ON PREDICTION OF METAL MELTING WITH THE AID OF A VOLUME HEAT SOURCE

A. A. Uglov and O. G. Sagdedinov

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Consideration is given to a process of solid metal target melting with the aid of an exponentially decreasing volume heat source. The problem is solved by the approximate analytical method with an error of about one per cent at temperatures up to values close to boiling points.

Solving the problems on melting of a target exposed to the action of energy fluxes absorbed on its surface is a matter of concern of many researchers [1-10]. In [11] and some other works consideration is given to an exponentially decreasing volume source without an account of melting heat, with the temperature field found in the form of an integral of a special function. In [12], in the form of special functions the authors present the solution for a stationary temperature field with ablation from an exponentially decreasing volume source. Usually in applications it is most important to determine the temperature field and the depth of the melting zone in dependence on thermophysical characteristics of the target and parameters of the emerging heat flux. For this, approximate analytical methods or numerical computations using one of the finite-difference schemes are employed.

In the present work we offer an approximate analytical solution to a melting problem in the case of an exponentially decreasing energy source with an account of melting heat absorption. Such an energy source arises, for instance, when some materials are exposed to laser radiation.

Let the constant energy flux be incident onto a solid target and produce an exponentially decreasing volume heat flux in it. We shall consider a region near the center of a heating spot until the condition $R \gg \sqrt{at}$ is fulfilled. Then the problem may be solved in the one-dimensional statement. Also, it is assumed that the absorbed density of the energy flux is larger than 10^9 W/m^2 and the temperature of the target surface does not exceed the boiling point of the metal. Then, as estimations show, heat losses from the target surface due to convection, radiation, and evaporation may be neglected as compared to the absorbed specific heat flux during melting and furthermore to the total energy flux density absorbed in the target. Once the target surface has been heated to the melting temperature, a near-surface melting layer is formed whose internal boundary of liquid and solid phases starts moving inside the target, while the external boundary remains stationary. The moving melting front undergoes absorption of the latent heat of a phase transition. Thermophysical characteristics of the metal and its melt are assumed equal and constant. Mathematically, the problem may be then written as follows:

$$
\begin{gathered}
\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial z^2} + \frac{Q}{c\gamma}\exp(-kz), \quad z \geqslant 0, \quad t \geqslant 0, \\
T(t=0) = T(z=\infty) = T_0, \quad \frac{\partial T}{\partial z}(z=0) = 0, \\
t \geqslant t_1: -\lambda\frac{\partial T}{\partial z}(z=s-0) = -\lambda\frac{\partial T}{\partial z}(z=s+0) + \\
+\gamma L\frac{ds}{dt}, \quad T(z=s) = T_m, \quad s(t_1) = 0.
\end{gathered} \tag{1}
$$

We introduce the dimensionless quantities

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$$z = zk, \quad s = sk, \quad t = tak^2, \quad T = \frac{T - T_0}{T_m - T_0},$$

$$A = \frac{Q}{c\gamma ak^2 (T_m - T_0)}, \qquad B = \frac{L}{2\sqrt{\pi}\, c\, (T_m - T_0)}.$$

Having used the function of the heat source [6] and taking approximately all the integrals analogously to [10], we arrive at the following expression for the temperature field:

$$T = A\left(f_1(t,\ z) - \exp(-z) + f_2(t,\ z)\right) - 2B\frac{s}{t - t_1} \times$$
$$\times \left(f_1(t - t_1,\ z - s) + f_1(t - t_1,\ z + s)\right), \quad t \geqslant t_1,$$
$$f_1(t;\ z) = 2\sqrt{\frac{t}{\pi}}\exp\left(-\frac{\sqrt{\pi}\,|z|}{2\sqrt{t}} - \frac{(\pi - 2)\, z^2}{8t}\right),$$
$$f_2(t,\ z) = \frac{1}{\sqrt{\pi}}\left(\exp(t + z)\left(\frac{\sqrt{\pi}}{2} - f_3\left(\left(1 + \frac{z}{2t}\right)\sqrt{t}\right)\right) + \right. \tag{2}$$
$$\left. + \exp(t - z)\left(\frac{\sqrt{\pi}}{2} - f_3\left(\left(1 - \frac{z}{2t}\right)\sqrt{t}\right)\right)\right),$$
$$f_3(y) = \int_0^y d\eta \exp(-\eta^2) = \alpha_1 y\left(\exp(-y^2) + \right.$$
$$\left. + \sqrt{\alpha_2 y^2 + \alpha_3 \exp(-2y^2)}\right)^{-1}, \quad \alpha_1 = \left(1 - \frac{3}{2\pi}\right)^{-1},$$
$$\alpha_2 = \frac{4\alpha_1^2}{\pi}, \quad \alpha_3 = (\alpha_1 - 1)^2.$$

The accuracy of the calculations for metals using (2) is about 1% of the maximum temperature value at the point $z = 0$. However, it includes the function of the melting zone depth $s(t)$, which has not been yet determined. Having assumed $z = s(t)$, $T(z = s(t), t) = 1$, we obtain the following transcendental equation for determination of $s(t)$:

$$1 = A\left(f_1(t,\ s) - \exp(-s) + f_2(t,\ s)\right) - \frac{2Bs}{t - t_1} \times$$
$$\times \left(f_1(t - t_1,\ 0) + f_1(t - t_1,\ 2s)\right), \quad t \geqslant t_1.$$

For metals, the parameter B is, as a rule, small. For instance, for titanium $B = 0.055$. The parameter A may acquire any values. To illustrate the method, calculations are made for titanium at $A = 1$. The results are compared to the case when absorption at the same energy contribution is superficial. Figure 1 depicts time dependences of the melting zone depth of the target obtained both with and without an account of absorption of the melting heat. It is seen that in the latter case the velocity of growth of the melting layer thickness at the moment of the onset of melting is infinite. In the case of superficial absorption the velocity of growth of the melting layer thickness at the moment of the onset of melting is finite in both cases. Besides, with volume absorption an account of the melting heat gives a more substantial correction for the time dependence of the melting layer thickness. As for the onset of melting in the case of volume absorption, it proceeds at a 2.5-fold longer time than in the case of superficial absorption. From Fig. 2 it is evident that in the case of volume absorption the temperature of the target surface increases considerably more slowly; furthermore at the moment of the onset of melting on volume absorption and with an account of the melting heat the rate of growth of the target surface temperature becomes equal to zero.

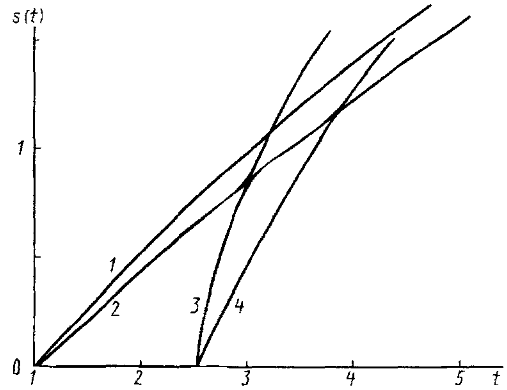

Fig. 1. Time dependences of the melting zone depth of the target for superficial [1) B = 0; 2) 0.055] and volume [3) A = 1; B = 0; 4) 1 and 0.055] absorption with the same energy contribution (dimensionless coordinates).

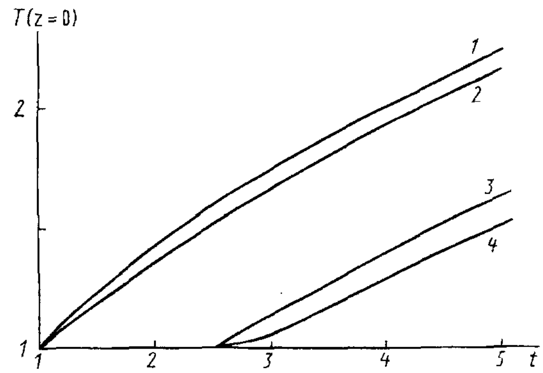

Fig. 2. Time dependences of the target surface temperature for superficial and volume absorption with the same energy contribution (dimensionless coordinates). Designations are the same as in Fig. 1.

NOTATION

t, time of energy flux action; t_1, moment of the onset of target melting; z, spatial coordinate counted from the target surface to its depth; T(t, z), temperature field in the target; T_m, melting temperature of the target; R, radius of transverse localisation of energy flux on the target surface; λ, target thermal conductivity; a, target thermal diffusivity; γ, density; c, specific heat; Q, volume power of a heat source on the target surface; k, concentration coefficient of a volume heat source; s(t), depth of target penetration; L, latent specific heat of melting; B, Stefan constant; A, free parameter of the problem.

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